Risk, Return, and Gambling Market Efficiency

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Adjusting for risk in the test of gambling market efficiency

Abstract

The current standard test of efficiency in gambling markets (that is, sports betting including horseracing) is that inefficiency exists when a betting strategy makes money. This level of economic significance is insufficient out-of-hand. Specifically, anyone can make money by buying a risk-free asset. We offer a new test of gambling efficiency that evaluates the risk and return of betting strategies relative to those of alternative investments. The method incorporates the well-documented Kelly criterion into the current method of investigation. The home-underdog bias of NFL games is reinvestigated with the tests indicating the bias is much less of an inefficiency than previously reported. An additional insight provided indicates a rationale for the favorite-longshot bias in horseracing.

Correcting for risk-adjusted returns in the test of gambling market efficiency

Mark Cuban, the flamboyant owner of the National Basketball Association's Dallas Mavericks, in a November 27th 2004 entry in his web blog, www.blogmaverick.com, stated that he was going to start a hedge fund and intends for sports betting to be the investment vehicle of the fund. Apparently, Mr. Cuban has been a hedge fund investment advisor in the past and now, through conversations and readings, believes that there are individuals with "smart money" abilities in betting. Mr. Cuban asks, "Can (a betting fund) significantly outperform the S&P as this new fund would be expected to do?" Mr. Cuban's question is interesting both from applied and academic settings. While only actual implementation of a fund would indicate a direct answer, we can offer an answer as to how successful the individual "investments" or bets would need to be to outperform the S&P 500. In so doing, we improve the method of efficiency testing in gambling markets and provide significant insight into the notion of sports gambling market inefficiency.

Currently, the simple gambling market test of market efficiency is to determine if significant profits are made by a betting strategy. That is, can we make money betting? This is inconsistent with basic financial economic theory of comparing the relevant risk and return among investments. In order to allow relative comparisons, we marry the widely known and well-documented Kelly criterion to the simple test. The result is a straightforward test of market efficiency that is consistent with accepted economic theory. Examples are provided and indicate the extent of the inadequacy of the current test. Examining well known "market inefficiencies" from US football and horseracing indicate that they would be better termed biases rather than inefficiencies.

The next section presents a brief review of gambling market basics. The third section reviews the literature and develops the new method of testing. The subsequent section presents some ad hoc levels of efficiency demonstrating the importance of the method followed by an examination of the well-known home-underdog NFL bias. A short conclusion follows.

III. Sports gambling markets

Most people assume that the long run outcome of point spread or horserace betting is no different from that of other casino games. That is, gamblers lose money in the end because the odds favor the "house." However, the effective odds for sports betting and horse racing are a direct result of human decisions and can therefore potentially exhibit consistent error. Bookmakers set betting lines and the pari-mutuel pools are set by the cumulative monies bet in the pools by individual bettors. That human decisions are integral in the outcomes of the bets rather than the fixed odds of dice throw or card play outcomes suggest that they may not be random and could represent solid investments opportunities. Reasons for this could include; "fixes" where players are paid to under perform (we know of those arrested but how many go undetected?), bookmakers consistently erring intentionally or unintentionally, and the more widely suspected and investigated consistent aggregate betting errors.

The major effort of academic research in gambling markets has been to determine if consistent, economically significant profits occur (often labeled as inefficiencies). In horseracing, the simple test is for the money winnings of a betting strategy to exceed a null of no profits – for example the money returns to a standard two-dollar bet on all horses going off at 1-1 or lower odds. In point spread betting such as on US football and

basketball, the common profit benchmark on sides, a team beats the opponent by so many points, or totals, the total points scored in the contest goes over or under a set number of total points, is a strategy producing a win percentage over 52.38 percent. Standard binomial statistical methods are used to test for the statistical significance of 52.38 percent. The 52.38 percent derives from the commonly used eleven-for-ten betting rule for sides and totals betting. A winning bet pays ten units of winnings for eleven units bet. To profit, a bettor must win more than, $11 / (10+11) \approx 52.38$ percent of her bets. We will use the sides and totals eleven-for-ten betting rule as a basis for our analysis.

a. The profit fallacy

The error of using mere profits to claim gambling inefficiency is shown out-of-hand. Anyone can make profits by simply opening an interest bearing account at a local financial institution. Simply put, a win percentage of 52.38 percent on bets is the level where point spread betting becomes a viable investment alternative rather than a gamble. Inefficiency can only be claimed if *abnormal* profits of an investment strategy are found after careful consideration of the risk associated with the strategy are compared with the relative risk and return of investment alternatives. According to Mr. Cuban, the relevant return of gambling strategies should be at least that of the SP 500. This is debatable and we will develop both sides of the debate and offer our suggestions shortly. However, at a minimum, gambling returns would need to better Treasury bill yields in order to offer viable investment alternatives. Thus, the win percentage of 52.38 percent is obviously inadequate.

The method we develop considers sample size and the relative riskiness of the strategy. While the test is developed using point spread betting, it is broadly applicable because it is easily adapted to other areas of interest such as horseracing.

III. Relevant Literature

Initially, Tryfos, Casey, Cook, Leger, and Pylypiak, (1984), showed that the test for point spread betting efficiency, rather than a simple Z-test against winning fifty percent of the time, must incorporate the eleven-for-ten betting rule. Woodland and Woodland (1997) modified the Tryfos et al. test to not include bets that end in ties. This method, a simple Z-test against a null win percentage of 52.38 percent, is the current standard of evaluation and has been used in some form by various authors to test for "inefficiencies" with mixed success. These include but are not be limited to; Vergin and Scriabin (1978), Zuber, Gandar, and Bowers (1985), Brajer, Ferris, and Marr (1988), Gandar, Zuber, O'Brien and Russo (1988), Lacey (1990), Golec and Tamarkin (1991), Brown and Sauer (1993), Badarinathi and Kochman (1996), Dare and MacDonald (1996), Gray and Gray (1997), Gandar, Dare, Brown, and Zuber (1998), Vergin and Sosik (1999), and Dare and Holland (2004). Obviously, this is an important and widely used method.

A first step beyond the simple test requires observation of the obvious. Gambling is inherently very risky with money bet at risk of complete, potentially instantaneous, loss. It is well known in the industry that a bettor risks financial ruin if they pursue a flat dollar betting strategy, which is frequently employed and the reason for so many financially troubled gamblers. Unfortunate runs of losing bets can eat up the entire stake quickly with flat betting so proportional betting is used. Proportional betting, where only

a proportion of wealth is bet on each bet, is the only strategy that avoids financial ruin. Any investigation of proportional betting strategies quickly leads one to the Kelly criterion.

a. The Kelly criterion

James L. Kelly Jr. was a mathematician that while working for Bell Labs determined the equation to allow engineers to minimize signal loss on noisy telephone lines. When published, others realized the far-reaching importance of the equation and adapted it to investments and gambling (primarily attributed to Thorpe (1975), see Ziemba and Hausch (1987)). The Kelly criterion (or a fractional Kelly strategy) is a proportional strategy shown by numerous authors as the optimum money management strategy for betting. These authors include; Breiman, Hakansson, and Thorp (1975), Bell and Cover (1980), Ethier and Tavare (1983), Finkelstein and Whitley (1981), Friedman (1981), Griffin (1984) and MaClean, Ziemba and Blazenko (1987).

The Kelly criterion gives the optimum proportion of wealth to bet on each bet of a strategy in order to maximize terminal wealth. No other strategy can give a higher growth rate of wealth, which makes it a good candidate to use in the investigation of efficiency. Mclean, Ziemba and Blazenko (1987) and McClean show that fractional Kelly strategies are "effective" in that betting a fixed fraction (for example a half or a quarter) of the Kelly proportion reduces return monotonically but curvilinearly reduces risk and therefore offers potentially interesting strategy tradeoffs. However, for our purposes, using the full Kelly proportion offers the highest return for a strategy and thus gives us the maximum likelihood of rejecting efficiency. However, this maximization property of the Kelly criterion leads to a large caveat as will be discussed later.

To use the Kelly criterion, we define:

w = win percentage of the strategy,
f = payoff ratio (amount paid per amount bet, which here is a constant 10/11 but can
      be set at other odds ratios as necessary for example those common in
      horseracing).

From the Kelly criterion, the optimum proportion of wealth to bet is:

$$p = w - \frac{(1-w)}{f} \qquad (1).$$

The growth factor of wealth per bet is:

$$g = (1+fp)^{w}(1-p)^{(1-w)} \qquad (2)$$

A description for (2) is that after subtracting one, (2) is the average expected increase in wealth per bet. The overall return to a strategy, for example, for a season of betting is:

$$r = g^{N} - 1, \qquad (3)$$

where, N is the number of bets made by the strategy.

Equation (3) expresses the optimum return to a betting strategy as a function of two unknowns, w and N, and a given constant, f. In examining ex-post betting strategy results, it is a simple matter to use the strategy win percentage w and the number of bets made by the strategy N to solve for the strategy return, r. The calculated return can be then be compared against alternative investments to determine the efficiency of a strategy. For example, Dare and Holland (2004) report that betting NFL home-underdogs for the six 1995-2000 seasons would have resulted in 374 bets with a win percentage of 55.6 percent. Plugging these values into (3) would yield a return of 117.93 percent for the six-year period – a respectable 13.86 percent per year return over the period.

However, using the inverse of this method should prove easier and more consistent with past methods of evaluation. That is, set *r* equal to the appropriate return level for the risk of a strategy and then solve for the win proportion *w* needed to attain that level of return. This win proportion becomes the appropriate hurdle rate, like the current level of 52.38 percent, but because *r* is set by the researcher, the method adjusts for the appropriate level of risk and return for the strategy under investigation. In the next section, we use this method of solving for win proportions *w* to develop new hurdle rates for various number of bets and return levels for strategies.

IV. The new betting strategy hurdle rate *w*

Table 1 contains Kelly criterion hurdle rates (*w*) for ad hoc bet numbers and levels of required returns. Two hundred bets represent a very broad strategy such as possibly betting all games in a season. The lower levels of ten bets through sixty bets represent refined strategies with fewer bets such as those investigated by many, such as Gray and Gray (1997) and Lacey (1990). The 52.38 percent method of analysis is well known and thus provides an established standard for comparison.

Table 1: Win proportions needed to claim inefficiency assuming an alternative investment return of the risk free rate as shown.

	Bets(*N*)					
R	200	60	50	40	20	10
3%	53.24%	53.95%	54.09%	54.30%	55.09%	56.21%
5%	53.48%	54.39%	54.59%	54.85%	55.86%	57.30%
9%	53.85%	55.05%	55.31%	55.65%	57.01%	58.91%
12%	54.06%	55.45%	55.74%	56.13%	57.68%	59.86%
15%	54.25%	55.78%	56.10%	56.55%	58.27%	60.69%

The clear message from Table 1 is that 52.38 percent is simply too low of a hurdle rate to use in the investigation of efficiency. For example, a common sample size for a

strategy would be 50 observations. If we assume a minimum required return of five percent, the strategy would need to statistically hurdle a 54.59 percent win proportion in order to conclude the strategy produced significant profits. That is quite a few more wins than the current standard of 52.38 percent and we still have not accounted for the risk of the strategy. If we use, as Mr. Cuban has suggested, the required return at a level expected on the S&P, say fifteen percent, we would need the strategy to best a win proportion of 56.10 percent. That is a rather high win percentage to maintain consistently and is difficult to requires a high win percentage to reject the null of efficiency.

As an example using actual data, we apply the new hurdle rate to the well-documented NFL home-underdog anomaly. Using NFL sides data from *Computer Sports World* and *The Goldsheet* for the period 1976-1999, there are an average 70 home-underdogs a season. Assuming a generously low six percent required return, the home-underdog strategy needs to hurdle 54.42 percent to claim inefficiency in any year. According to the data, that occurred on a statistically significant basis in only one of the 24 years between 1976 and 1999 (1978 with a 68.8 win proportion). If we aggregate the games, there are 1677 home-underdog games for the period. Again using a six percent required return to solve for *w*, the hurdle rate goes to 52.8 percent. The home-underdog strategy produces a win percentage of 54.68 percent and is not marginally significant (t-value of 1.5469) although it is when tested against 52.38 percent (t-value of 1.8923). In other words, the home-underdog anomaly cannot be considered an inefficiency and would best be labeled a bias.

a. Extending the new hurdle rate to horseracing

It is a simple matter to develop new hurdle rates for other gambling strategies such as horseracing. Horseracing involves differing odds levels and so to find the win proportion necessary for efficiency the appropriate odds ratio f in (1) and (2) is used and w calculated for the required return. Table 2 contains the hurdle rate win proportions for 50 bets at increasing levels of returns and differing odds levels.

Table 2: Win proportions needed per 50 bets in order to claim inefficiency at differing bet odds at different investment returns.

Return/ 50 bets	Odds							
	1-5	1-2	1-1	3-1	6-1	10-1	15-1	30-1
0%	0.8333	0.6667	0.5000	0.2500	0.1429	0.0909	0.0625	0.0323
3%	0.8460	0.6828	0.5172	0.2650	0.1550	0.1010	0.0710	0.0385
5%	0.8496	0.6874	0.5221	0.2693	0.1585	0.1039	0.0735	0.0404
9%	0.8548	0.6941	0.5293	0.2757	0.1638	0.1082	0.0772	0.0432
12%	0.8579	0.6981	0.5337	0.2795	0.1669	0.1109	0.0794	0.0448

Again, it is easy to see that increasing levels of return require greater win proportions from a strategy in order to produce abnormal profits. For a 1-5 favorite bet, a breakeven win proportion of 0.8333 increases to 0.8579 at a twelve percent return. This is more than two winning bets per 100 bets placed. However, for 30-1 long shot bets, the breakeven win proportion increases from 0.0323 to 0.0448, just slightly more than one win in 100 bets. This is an interesting insight. The number of additional wins to achieve abnormal profits is less for long shot bets than for short favorite bets by nearly a half in this example. The significance of this is that a bettor only needs to find slightly more than one additional winner per 100 bets for long shot strategies to produce significant profits but would need to find more than two additional winners per 100 in picking short

money favorites. That seems like twice the work and could explain why short money favorites are typically underbet.

V. The relevant return

With the method of statistical investigation in place, we turn to the important question of which return to use. Gandar, Zuber, O'Brien and Russo (1988) suggest that betting has no systematic risk. This assumption seems reasonable; however, even zero beta investments should return the risk-free rate. We argue that the risk-free rate is insufficient because of the enormous undiversifiable risk betting strategies have if using a Kelly strategy. Strategy risk is not the result of the result of bet covariance. There should be little covariance between bets unless the strategy on both teams in a contest or more than one horse in a race. Simultaneously, there may be some covariance of bets on, for example, the same team during a season or the same horse over many races. For our purposes, we will assume zero correlation between wagers.

The problem with diversification of betting strategies lies in the fact that following a Kelly strategy results in large wagers of wealth on all or nothing outcomes. Take a strategy that bets NFL sides and is expected to win 56 percent of the bets. Using (1) we find that the optimum amount to bet is $0.56 – (1-.56) / (10/11) = 0.076$ of wealth on each bet. That means on a Sunday afternoon in the fall, a bettor might have four bets placed on NFL games with each bet worth 7.6 percent of wealth. That represents 30.4 percent of wealth wagered on outcomes that could all potentially lose the entire amount bet. If the strategy truly produces 56 percent winners, there is a 3.75 percent chance (($1-0.56)^4$) of that that catastrophe occurring! That is worse than any recorded one-day market crash and certainly not what any rational investor would call diversified. On

more probable winners such as a strategy of betting 1-5 favorites at the track would require a bet of sixteen percent of wealth per bet with an approximately ten to fifteen percent chance you would lose the entire amount. In other words, there is enormous risk in betting strategies that cannot be diversified and therefore requires much more required return than the risk-free rate and we contend that due to enormous undiversifiable risk, it should be much more than the S&P 500.[1]

So what would seem like a reasonable return to use? We suggest something really large – like 30 to 40 percent. At a 40 percent required return, a bettor of sides or totals would need to achieve a 58.16 winning percentage to claim inefficiency. At the same required return, a horserace strategy of betting 1-5 favorites would need to win 87.5 percent of all bets to claim inefficiency. This win level is well above the 83.33 percent needed to breakeven.

VI. Additional Caveat

The issue of undiversifiable risk is no small item. However, an additional problem for a claim of inefficiency arises in that a Kelly strategy assumes that the bettor knows the correct win proportions of a given strategy. The Kelly criterion gives the *optimum* level to bet. If more or less than the optimum is bet, then the strategy will not produce the expected level of return. This means that a researcher would need to assume that the bettor would know beforehand exactly the amount to bet in order to attain the return. But that's a little like picking horses at the track isn't it? In other words, the method outlined is biased in favor of rejection of the efficiency null hypothesis in that it is unlikely that the bettor would ever attain the returns given in (3). Again, this suggests

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[1] A possible way to reduce strategy risk is to use a fractional Kelly strategy, however, this proportionally reduces returns and therefore does not alleviate the situation.

a higher level of return should be used in the investigation of efficiency. We leave it to further research to determine what these appropriate levels might be.

VII. Conclusion

The test for betting market efficiency was rejected out-of-hand because of it's erroneous hurdle rate of simply making a profit. At a minimum, a betting strategy should at least make the risk-free rate to be considered an inefficiency. A method to determine if a strategy bests a reasonable required return for risky assets is developed. The method uses the Kelly criterion to calculate new hurdle rates for efficiency. We calculate new hurdle rates for reasonable numbers of bets and required returns to allow comparison to the current 52.38 percent level used to judge point spread betting market efficiency. It is shown that 52.38 percent is an inadequate level to deem inefficiency. Additional hurdle rates were calculated to show the applicability of the method to odds markets such as horseracing.

Using the new hurdle rates on the NFL home-underdog bias indicate that only one year in the last twenty-four was likely inefficient – about what one would expect to find in a random sample. Over the entire sample, we did not find statistical significance for inefficiency of a strategy of betting the home underdog using the new hurdle rate.

Finally, we argued that a risk-free hurdle rate of return for gambling markets is woefully inadequate and offer a proposed level, while not completely quantified, that is at least double that of the market return and could be approaching 30-40 percent. The proposed method has broad applicability and "real world" assumptions. It should be used in all future investigations of gambling market efficiency.

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